Lecture 4 summary:

Basic reservoir simulation concepts - flow and displacement concepts.

2.6. Multiphase Flow Concepts

Interfacial Tension

Interfacial tension (IFT) is the tension between liquids at a liquid-liquid interface.

Surface tension refers to the tension between fluids at a gas-liquid interface.

Interfacial tension is energy per unit of surface area, or force per unit length. The value of IFT depends on the composition of the two fluids at the interface between phases. Table 6 lists a few examples:

Table 6. Examples of Interfacial Tension

Fluid Pair	IFT Range (mN/m or dyne/cm)
Air-Brine	72-100
Oil-Brine	15-40
Gas-Oil	35-65

Wettability:

Wettability is the ability of a fluid phase to wet a solid surface preferentially in the presence of a second immiscible phase.

The wettability condition in a rock/fluid system depends on IFT. Adding a chemical such as surfactant, polymer, corrosion inhibitor, or scale inhibitor can alter wettability.

Wettability is measured by contact angle. Contact angle is always measured through the denser phase and is related to interfacial energies by:

$$\sigma_{os} - \sigma_{ws} = \sigma_{ow} \cos\theta, \quad \text{-----------------------------------} \quad (4)$$

where

σ_{os} - interfacial energy between oil and solid (dyne/cm)

σ_{ws} - interfacial energy between water and solid (dyne/cm)

σ_{ow} - interfacial energy, or IFT, between oil and water (dyne/cm)

θ - contact angle at oil-water-solid interface measured through the water phase (degrees)

Table 7 presents examples of contact angle for different wetting conditions.

Table 7. Examples of Contact Angle

Wetting Condition	**Contact Angle (Degrees)**
Strongly water-wet	0-30
Moderately water-wet	30-75
Neutrally wet	75-105
Moderately oil-wet	105-150
Strongly oil-wet	150-180

Capillary Pressure

Capillary pressure is the pressure difference across the curved interface formed by two immiscible fluids in a small capillary tube. The pressure difference is

$$P_c = P_{nw} - P_w \quad , \qquad (5)$$

Where,

P_c - capillary pressure (psi)

P_{nw} - pressure in nonwetting phase (psi)

P_w - pressure in wetting phase (psi)

Equilibrium between fluid phases in a capillary tube is satisfied by the relationship *force up* = *force down*.

Equivalent Height

The equivalent height is the height above the free fluid level of the wetting phase, where the free fluid level is the elevation of the wetting phase at P_c = 0. Relationship between capillary pressure P_c and equivalent height h is:

$$h = \frac{P_c}{(\Gamma_w - \Gamma_{air})} \quad , \qquad (6)$$

where:

h height of capillary rise (ft)

P_c capillary pressure (psia)

Γ_w - water, or wetting phase, density gradient (psia/ft)

Γ_{air} - air, or non wetting phase, density gradient (psia/ft)

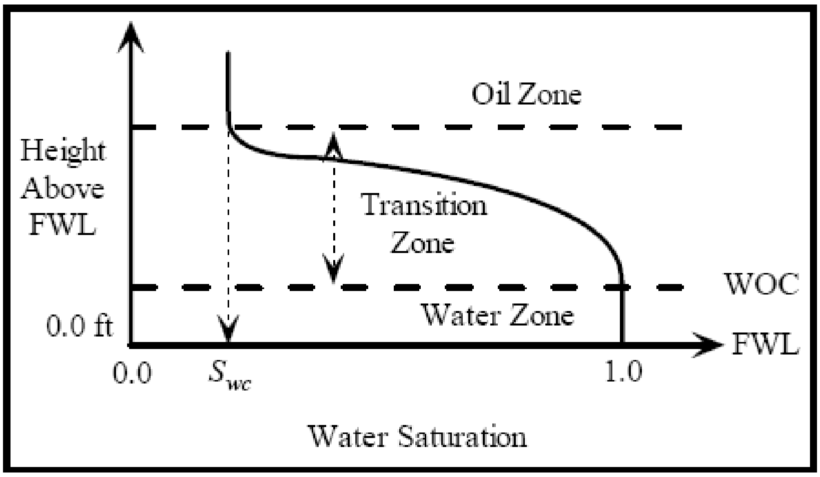

Figure 7. Sketch of an Oil-Water Transition Zone

(Note that WOC is water-oil contact, FWL is free water level)

Equivalent height is inversely proportional to the difference in densities between two immiscible phases.

The preceding definitions of free fluid level and fluid contact are based on capillary pressure.

The modeling team should know how free fluid levels and fluid contacts are defined to avoid confusion.

Leverett's *J*-Function

Leverett's *J*-function is a technique for correlating capillary pressure to water saturation and rock properties. Leverett's *J*-function is:

$$J(S_w) = \frac{P_{c(lab)}}{\sigma_{lab}\left|(\cos\theta)_{lab}\right|}\left(\sqrt{\frac{K}{\phi}}\right)_{lab} \quad \text{, ---------------------------------- (7)}$$

Where,

$P_{c(lab)}$ - Laboratory measured capillary pressure (psia)

$J(S_w)$ - Leverett's *J*-function

K - Core sample permeability (md)

ϕ- Porosity (fraction)

σ_{lab} Laboratory value of IFT (dyne/cm)

θ_{lab} - Laboratory value of contact angle

Given $J(S_w)$, we can estimate capillary pressure at reservoir conditions as:

$$P_{c(res)} = \frac{\sigma_{res} \left| (\cos\theta)_{res} \right|}{\left(\sqrt{\frac{K}{\phi}} \right)_{res}} J(S_w)$$, --- (8)

Relative Permeability

Relative permeability is used to describe multiphase fluid flow.

Fluid phase relative permeabilities for oil, water and gas phases, respectively, are

$k_{ro} = k_o / k$, $k_{rw} = k_w / k$, $k_{rg} = k_g / k$, ------------- (9)

where,

k_o , k_w , k_g - the effective permeability of oil, water and gas respectively.

k - absolute permeability(md).

Figure 8 shows a typical set of relative permeability curves.

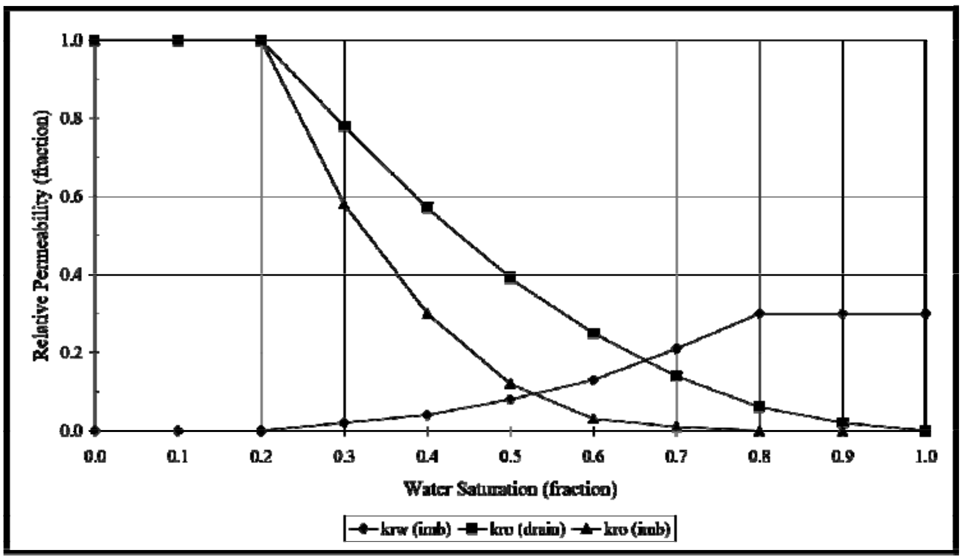

Figure 8. Typical Water-Oil Relative Permeability Curves

Mobility

Mobility is a measure of the ability of a fluid to move through interconnected pore space.

Fluid phase mobility is the ratio of effective phase permeability to phase viscosity. Mobility for oil, water and gas phases respectively are

$$\lambda_o = \frac{k_o}{\mu_o}, \lambda_w = \frac{k_w}{\mu_w}, \lambda_g = \frac{k_g}{\mu_g} \text{, --- (10)}$$

Where, μ is the viscosity of phase.

Relative mobility is defined as relative permeability divided by viscosity.

Mobility ratio is the mobility of the displacing fluid divided by the mobility of the displaced fluid.

Mobility ratio of water to oil:

$$M_{w,o} = \frac{(\lambda_w)_{S_{or}}}{(\lambda_o)_{S_{wc}}} = \frac{k_{rw}(S_{or})/\mu_w}{k_{ro}(S_{wc})/\mu_o} \text{, ------------------------------------ (11)}$$

In this case, relative permeability to water is evaluated at residual oil saturation S_{or}, and relative permeability to oil is evaluated at connate water saturation S_{wc}. Notice that absolute permeability factors out of the expression for mobility ratio. Consequently, mobility ratio can be calculated using either mobilities or relative mobilities.

Fractional Flow

Fractional flow is the ratio of the volume of one phase flowing to the total volume flowing in a multiphase system.

The fractional flow of water is the ratio of water production rate to total production rate. The fractional flow of water is given by:

$$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_w + q_o} \text{, -- (12)}$$

where, f_w fractional flow of water. q_w water volumetric flow rate (RB). q_o oil volumetric flow rate (RB) q_t total volumetric flow rate (RB). Notice that the flow rates are expressed in terms of reservoir volumes.

The fractional flow of oil f_o and the fractional flow of water are related by $f_w = 1 - f_o$ for an oil-water system. Based on the definition of fractional flow, we see that fractional flow should have a value between 0 and 1.

If we in Darcy's Law neglect gravity and capillary pressure, Equation (12) can be expressed in terms of mobilities as

$$f_w = \frac{1}{1+\frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}} = \frac{1}{1+\frac{\lambda_o}{\lambda_w}}, \quad \text{--} \quad (13)$$

The construction of Eq. (13) is based on the following simplifying assumptions: Darcy's Law adequately describes flow rate, and capillary pressure and gravity are negligible (Most waterfloods have sufficiently high flow rates that capillary pressure and gravity effects can be neglected). Given these assumptions, we can calculate f_w at reservoir conditions.

2.7. Fluid Displacement Concepts

Buckley- Leverett Theory

The movement of the interface between displacing and displaced fluids and the breakthrough time are indicators of sweep efficiency.

Buckley-Leverett method [1942] is the simplest method of estimating the advance of a fluid displacement front. It is an application of the law of conservation of mass and can be expressed as (Figure 9):

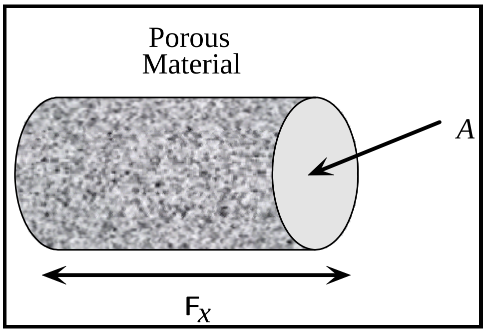

Figure 9. Flow Geometry

$$x_{S_w} = \frac{W_i}{A\phi}\left(\frac{df_w}{dS_w}\right)_{S_w}$$, -------------------------------------- (14)

Where,

x_{Sw} - distance traveled by a particular S_w contour (ft).

W_i - cumulative water injected (cu ft).

$(df_w/dS_w)_{Sw}$ – change in fractional flow divided by change in saturation (slope of fractional flow curve).

Water Saturation Profile

A plot of S_w versus distance using Eq. (14) and typical fractional flow curves leads to the physically impossible situation of multiple values of S_w at a given location.

Welge's Method is widely used to perform the Buckley-Leverett frontal advance calculation. Welge's approach is best demonstrated using a plot of f_w versus S_w (Figure 10).

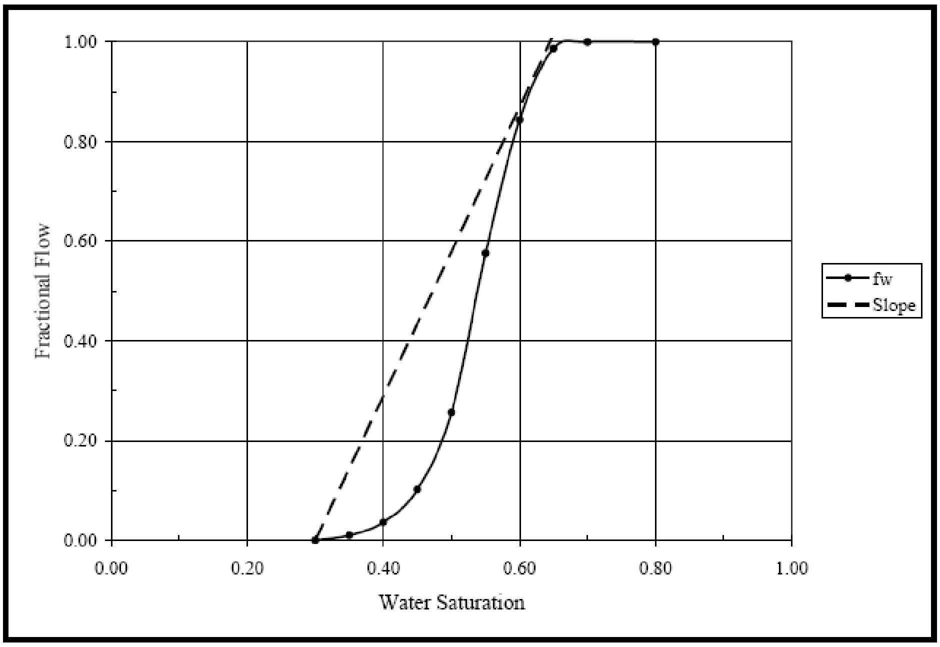

Figure 10. Welge's Method

A line is drawn with its intercept at the irreducible water saturation S_{wirr} – the water saturation S_w in front of the waterflood – and tangent to a point on the f_w curve. The resulting tangent line is called the breakthrough tangent, or slope. Water saturation at the flood front S_{wf} is the point of tangency on the f_w curve.

The water-oil flood front is sometimes called a shock front because of the abrupt change from irreducible water saturation in front of the waterflood and S_{wf}. Fractional flow of water at the flood front is f_{wf} and occurs at the point of tangency S_{wf} on the f_w curve. In Figure 9, S_{wf} is 62% and f_{wf} is 92. Average water saturation behind the flood front S_{wbt} is the intercept of the main tangent line with the upper limiting line where f_w = 1.0.

In Figure 6, average S_{wbt} is 65%. In summary, when injected water reaches the producer, Welge's approach gives the following results:

1. Water saturation at the producing well is S_{wf}
2. Average water saturation behind the front is S_{wbt}

3. Producing water cut at reservoir conditions is f_{wf}

Welge's approach can be used also to obtain the time to water breakthrough at the producer:

$$t_{bt} = \frac{LA\phi}{q_i \left(df_w / dS_w \right)_{S_w}} \quad , \text{--- (15)}$$

Where,

q_i - injection rate

$(df_w / dS_w)_{Sw}$ - slope of main tangent line

L - linear distance from injection well to production well

In the absence of capillary pressure and gravity effects, the flood front propagates as a relatively "sharp" step function, or piston-like displacement.

The presence of capillary pressure leads to the imbibition of water ahead of the front. The WOR will begin to increase sooner than it would have in the absence of capillary pressure. By contrast, gravity causes high S_w values to lag behind the front. The result is a smeared or "dispersed" flood front.

Viscous Fingering

If a low viscosity fluid is injected into a cell containing a high viscosity fluid, the low viscosity fluid will begin to form fingers as it moves through the fluid. These fingers can have different shapes depending on medium heterogeneity.

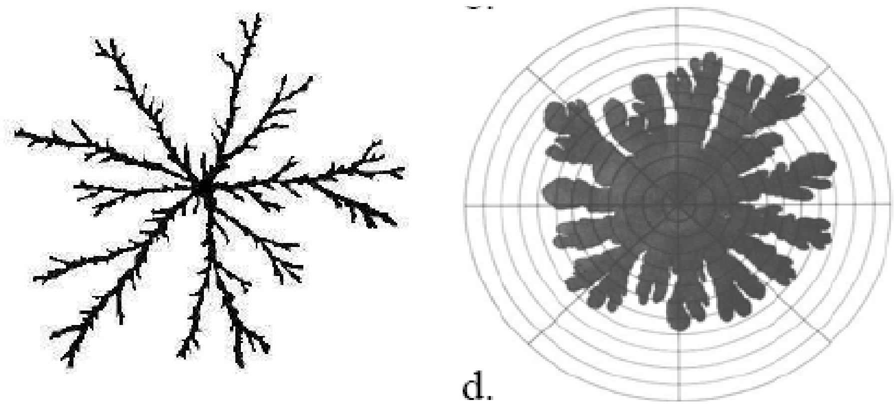

Figure 11. Viscous Finger

Most reservoir simulators do not accurately model fingering effects. It is possible to improve model accuracy by using a very fine grid to cover the area of interest, but the benefits associated with such a fine grid are seldom sufficient to justify the additional cost.